Find the arclength of the parametric curve $\begin{aligned} x &= t^3 + 2t^2 - 1 \\ y &= 2t^3 - t^2 \end{aligned}$ for $t \in [0, 1]$.

SCORE: _____ / 6 PTS

$$\int_0^1 \sqrt{(3t^2+4t)^2+(6t^2-2t)^2}\,dt \quad 2$$

$$= \int_0^1 \sqrt{9t^4 + \cancel{24t^3} + 16t^2 + 36t^4 - \cancel{24t^3} + 4t^2}\,dt$$

$$= \int_0^1 \sqrt{45t^4+20t^2}\,dt \quad 1$$

$$= \int_0^1 t\sqrt{45t^2+20}\,dt \quad 1$$

$$u = 45t^2+20 \quad \begin{aligned} t=1 &\rightarrow u=65 \\ t=0 &\rightarrow u=20 \end{aligned}$$

$$du = 90t\,dt \rightarrow t\,dt = \frac{1}{90}du$$

$$= \frac{1}{90}\int_{20}^{65} u^{\frac{1}{2}}\,du \quad \frac{1}{2} \quad \frac{1}{2}$$

$$= \frac{1}{\cancel{90}_{45}} \cdot \frac{\cancel{2}}{3} u^{\frac{3}{2}} \Big|_{20}^{65} \quad \frac{1}{2}$$

$$= \frac{1}{135}\left(65\sqrt{65} - 20\sqrt{20}\right) \quad \frac{1}{2}$$

$$= \frac{1}{27}\left(13\sqrt{65} - 8\sqrt{5}\right)$$

In a study of food waste, students at a school cafeteria were each given a 4 ounce portion of vegetables as part of lunch. Students were randomly selected, and X is the random variable representing the amount of the vegetable the student left behind (measured in ounces). Find the mean (average) amount of food wasted per student if the probability density function is given by

SCORE: _____ / 10 PTS

$$f(x) = \begin{cases} k(2+x)(4-x), & x \in [0, 4] \\ 0, & x \notin [0, 4] \end{cases}$$ (for some appropriate constant k).

$\underbrace{\int_0^4 k(8+2x-x^2)\,dx = 1}_{}$ 2

$k = \dfrac{1}{\underbrace{(8x+x^2-\frac{1}{3}x^3)\Big|_0^4}_{}}$ $1\frac{1}{2}$

$= \dfrac{1}{32+16-\frac{64}{3}}$

$= \underbrace{\dfrac{3}{80}}_{}$ $1\frac{1}{2}$

$\text{MEAN} = \underbrace{\int_0^4 \frac{3}{80} x(8+2x-x^2)\,dx}_{}$ 2

$= \frac{3}{80} \int_0^4 (8x+2x^2-x^3)\,dx$

$= \frac{3}{80} \left[\underbrace{4x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4}_{} \right] \Big|_0^4$ $1\frac{1}{2}$

$= \frac{3}{80} \left[\cancel{64} + \frac{128}{3} - \cancel{64} \right]$

$= \underbrace{\frac{8}{5} \text{ OUNCES}}_{}$ $1\frac{1}{2}$

For the function $f(x) = \dfrac{x}{\sqrt{25 - x^2}}$ on the interval $x \in [0, 4]$, find the value of c such that $f_{ave} = f(c)$. **SCORE: _____ / 7 PTS**

NOTE: This is the value c guaranteed by the Mean Value Theorem for Integrals.

$$f_{AVE} = \boxed{\dfrac{\int_0^4 \frac{x}{\sqrt{25-x^2}}\,dx}{4-0}}$$

$u = 25 - x^2$, $x=4 \longrightarrow u=9$, $x=0 \longrightarrow u=25$

$du = -2x\,dx \longrightarrow x\,dx = -\tfrac{1}{2}du$

$$= \underbrace{\tfrac{1}{4} \cdot -\tfrac{1}{2}\int_{25}^{9} \tfrac{1}{\sqrt{u}}\,du}$$

$$= -\tfrac{1}{8} \cdot \underbrace{2\sqrt{u}}\Big|_{25}^{9}$$

$$= -\tfrac{1}{4}(3-5)$$

$$= \underbrace{\tfrac{1}{2}}$$

1 POINT FOR EACH ITEM

$$\underbrace{f(c) = \frac{c}{\sqrt{25-c^2}} = \frac{1}{2}}$$

$$2c = \sqrt{25-c^2}$$

$$4c^2 = 25 - c^2$$

$$c^2 = 5$$

$$\underbrace{c = \sqrt{5}} \in [0, 4]$$

ONLY $\frac{1}{2}$ IF YOU INCLUDED $c = -\sqrt{5}$

Find the surface area if the curve $y=\sqrt{25-x^2}$ for $x \in [3, 5]$ is revolved around the $y-$axis. **SCORE: _____ / 7 PTS**

$$x=\sqrt{25-y^2} \longrightarrow \frac{dx}{dy}=\frac{1}{2}(25-y^2)^{-\frac{1}{2}}\cdot(-2y)=\frac{-y}{\sqrt{25-y^2}}$$

$$x\in[3,5] \longrightarrow y\in[0,4]$$

$$\underbrace{2\pi}_{1}\underbrace{\int_0^4}_{1}\underbrace{\sqrt{25-y^2}}_{1}\cdot\underbrace{\sqrt{1+\left(\frac{-y}{\sqrt{25-y^2}}\right)^2}}_{2}\,dy$$

$$=2\pi\int_0^4\sqrt{25-y^2}\sqrt{1+\frac{y^2}{25-y^2}}\,dy$$

$$=2\pi\int_0^4\sqrt{25-y^2}\sqrt{\frac{25}{25-y^2}}\,dy$$

$$=\underbrace{2\pi\int_0^4 5\,dy}_{1}$$

$$=10\pi(4-0)=\underbrace{40\pi}_{1}$$

NOTE: dx INTEGRAL NOT POSSIBLE SINCE $\frac{dy}{dx}$ UNDEFINED / NOT CONTINUOUS @ $x=5$